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SU(3)-breaking effects in kaon and hyperon semileptonic decays from lattice QCD

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Abstract. We present a quenched lattice study of the vector form factors at zero-momentum transfer, $f_V(0)$, relevant for the determination of $|V_{us}|$ from semileptonic $K \to \pi\ell\nu$ ($K_{\ell 3}$) and $\Sigma^- \to n\ell\nu$ decays. Using suitable double ratios of three-point correlation functions we show that in case of both kaon and hyperon decays it is possible to calculate the form factor $f_V(0)$ at the percent precision, which is the one required for a significant determination of $|V_{us}|$. In case of $K_{\ell 3}$ decay the leading quenched chiral logarithms are corrected for by using analytic calculations in quenched chiral perturbation theory. Our final result, $f_V(K^0\pi^-) = 0.960 \pm 0.005_{\rm stat} \pm 0.007_{\rm syst}$, where the systematic error does not include the residual quenched effects, is in good agreement with the estimate made by Leutwyler and Roos. The impact of our result on the extraction of $|V_{us}|$ is briefly illustrated.

PACS. 12.15.Hh Determination of CKM matrix elements – 13.20.Eb Semileptonic decays of K mesons – 13.30.Ce Semileptonic decays of baryons – 12.38.Gc Lattice QCD calculations

1 Introduction

The most precise determinations of the CKM matrix element $|V_{us}|$ [1] are presently obtained from the semileptonic weak decays of kaons and hyperons. The analysis of the experimental data on $K_{\ell 3}$ [2] and hyperon [3] decays can give access to the quantity $|V_{us}| \cdot f_V(0)$, where $f_V(0)$ is the vector form factor at zero-momentum transfer for each decay. A good theoretical control on these transitions is obtained via the Ademollo-Gatto (AG) theorem [4], which states that $f_V(0)$ is renormalized only by terms of at least second order in the breaking of the SU(3)-flavor symmetry. The estimate of the difference of $f_V(0)$ from its SU(3)-symmetric value is presently the dominant source of theoretical uncertainty in the extraction of $|V_{us}|$ and it will become the dominant one, when the results of high-statistics experiments, like KLOE and NA48 for $K_{\ell 3}$ decays, will be available.

The amount of SU(3) breaking due to light quark masses can be investigated within Chiral Perturbation Theory (CHPT) by performing a systematic expansion of the type $f_V(0) = 1 + f_2 + f_4 + \ldots$, where $f_n = \mathcal{O}[M^n_{K,\pi}/(4\pi f_\pi)^n]$. However, only f_2 can be computed unambiguously, while the higher-order terms involve unknown coefficients of several chiral local operators. Up to now their numerical impact has been estimated using phenomenological models, like the famous Leutwyler and Roos (LR) calculation carried out for $K_{\ell 3}$ decays using the constituent quark model [5].

Very recently [6] the SU(3)-breaking effects on $f_V(0)$ for $K_{\ell 3}$ decays have been estimated using a non-perturbative approach based on the fundamental theory. In [6] a new strategy has been proposed and successfully applied to $K_{\ell 3}$ decays in order to reach the challenging goal of a $\approx 1\%$ error on $f_V(0)$ using lattice QCD simulations.

The aim of this contribution is twofold. First, we illustrate both the basic features of the procedure of [6] and the quenched lattice results obtained for $K_{\ell 3}$ decays. Second, we present the results of an exploratory study of the application of the same strategy to hyperon decays [7], showing that the same level of precision reached for $K_{\ell 3}$ decays is attainable also in case of hyperons.

2 Semileptonic $K \to \pi\ell\nu$ decay

The $K^0 \to \pi^-$ form factors of the weak vector current $V_\mu = \bar{s}\gamma_\mu u$ are defined as

$$\langle \pi(p')|V_\mu|K(p)\rangle = f_V(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu \ , \quad (1)$$

where $q = p - p'$. As usual, we express $f_-(q^2)$ in terms of the so-called scalar form factor

$$f_0(q^2) \equiv f_V(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2) \ , \tag{2}$$

so that by construction $f_0(0) = f_V(0)$.

The procedure developed in [6] to reach the challenging goal of a $\approx 1\%$ error on $f_V(0)$, is based on the following three main steps:

1) precise evaluation of the scalar form factor $f_0(q^2)$ at $q^2 = q^2_{\rm max} = (M_K - M_\pi)^2$;
2) extrapolation of $f_0(q^2_{\rm max})$ to $f_0(0) = f_V(0)$;
3) subtraction of leading chiral logs and extrapolation of $f_V(0)$ to the physical meson masses.

These steps are described in the next three subsections, while the details of the simulation can be found in [6].

2.1 Evaluation of $f_0(q^2)$ at $q^2 = q^2_{\rm max} = (M_K - M_\pi)^2$

Following a procedure originally proposed in [8] to study heavy-light form factors, the scalar form factor $f_0(q^2)$ can be calculated very efficiently at $q^2 = q^2_{\rm max} = (M_K - M_\pi)^2$ (i.e. $\mathbf{p} = \mathbf{p}' = \mathbf{q} = 0$) from the double ratio of three-point correlation functions with both mesons at rest:

$$R_0(t_x, t_y) \equiv \frac{C_0^{K\pi}(t_x, t_y, \mathbf{0}, \mathbf{0})\, C_0^{\pi K}(t_x, t_y, \mathbf{0}, \mathbf{0})}{C_0^{KK}(t_x, t_y, \mathbf{0}, \mathbf{0})\, C_0^{\pi\pi}(t_x, t_y, \mathbf{0}, \mathbf{0})} \ , \tag{3}$$

where the three-point correlation function for the $K \to \pi$ transition is defined as

$$C_\mu^{K\pi}(t_x, t_y, \mathbf{p}, \mathbf{p}') = \sum_{\mathbf{x},\mathbf{y}} \langle O_\pi(t_y, \mathbf{y})\ \widehat{V}_\mu(t_x, \mathbf{x})\ O_K^\dagger(0) \rangle \cdot e^{-i\mathbf{p}\cdot\mathbf{x} + i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} \tag{4}$$

with $\widehat{V}_\mu$ being the renormalized lattice vector current and $O_\pi^\dagger = \bar{d}\gamma_5 u$, $O_K^\dagger = \bar{d}\gamma_5 s$ the meson interpolating fields.

When the vector current and the two interpolating fields are separated far enough from each other, the contribution of the ground states dominates, yielding

$$R_0(t_x, t_y) \xrightarrow[(t_y - t_x) \to \infty]{t_x \to \infty} \frac{\langle \pi | \bar{s}\gamma_0 u | K \rangle\, \langle K | \bar{u}\gamma_0 s | \pi \rangle}{\langle K | \bar{s}\gamma_0 s | K \rangle\, \langle \pi | \bar{u}\gamma_0 u | \pi \rangle} = [f_0(q^2_{\rm max})]^2\, \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} \ . \tag{5}$$

There are several crucial advantages in the use of the double ratio (3). First, statistical uncertainties are largely reduced, because fluctuations in the numerator and the denominator are highly correlated. Second, the matrix elements of the meson sources cancel between numerator and denominator. Third, R_0 is independent from both the improved renormalization constant and the $\mathcal{O}(a)$-improvement coefficient of the vector current, where a is the lattice spacing. Therefore R_0 suffers from discretization effects only at order a^2. Finally, $R_0 = 1$ in the SU(3)-symmetric limit at all orders in a. Thus the deviation of R_0 from unity depends on the physical SU(3)-breaking effects on $f_0(q^2_{\rm max})$ as well as on discretization errors, which are at least of order $a^2(m_s - m_\ell)^2$. Our result for $f_0(q^2_{\rm max})$ is not affected by the whole discretization error on the three-point correlation function, but only by its smaller SU(3)-breaking part. Similar considerations apply also to the quenching error, because $R_0 = 1$ in the SU(3)-symmetric limit holds as well also within the quenched approximation.

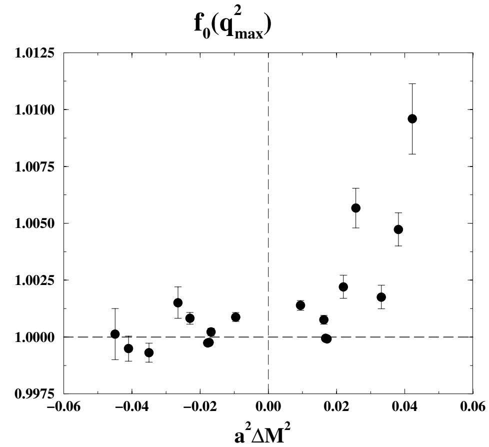

Fig. 1. Values of $f_0(q^2_{\rm max})$, obtained in [6] using (5), versus the SU(3)-breaking parameter $a^2\Delta M^2 \equiv a^2(M_K^2 - M_\pi^2)$.

From the plateaux of the double ratio (5) the values of $f_0(q^2_{\rm max})$ can be determined with an uncertainty smaller than 0.1%, as it is illustrated in Fig. 1.

2.2 Extrapolation of $f_0(q^2_{\rm max})$ to $f_0(0) = f_V(0)$

The extrapolation of $f_0(q^2)$ from $q^2_{\rm max}$ to $q^2 = 0$ requires the knowledge of the slope of f_0, which in turn implies the study of the q^2-dependence of the scalar form factor. An important remark is that in order to get $f_0(0)$ at the percent level the precision required for the slope can be much lower ($\simeq 30\%$), because the values of $q^2_{\rm max}$ can be chosen very close to $q^2 = 0$ [6].

For each set of quark masses two- and three-point correlation functions can be calculated for mesons with various momenta in order to extract the q^2 dependence of both $f_0(q^2)$ and $f_V(q^2)$. The latter turns out to be well determined on the lattice with a statistical error of $\simeq 5 \div 20\%$ and its q^2-dependence is very well described by a pole-dominance fit, $f_V(q^2) = f_V(0)/[1 - \lambda_V\ q^2]$. On the contrary for the scalar form factor the uncertainties turn out to be about 5 times larger. As explained in [6] the precision in the extraction of $f_0(q^2)$ can be drastically improved by constructing new suitable double ratios

$$R_i(t_x, t_y) = \frac{C_i^{K\pi}(t_x, t_y, \mathbf{p}, \mathbf{p}')}{C_0^{K\pi}(t_x, t_y, \mathbf{p}, \mathbf{p}')}\ \frac{C_0^{KK}(t_x, t_y, \mathbf{p}, \mathbf{p}')}{C_i^{KK}(t_x, t_y, \mathbf{p}, \mathbf{p}')} \ , \tag{6}$$

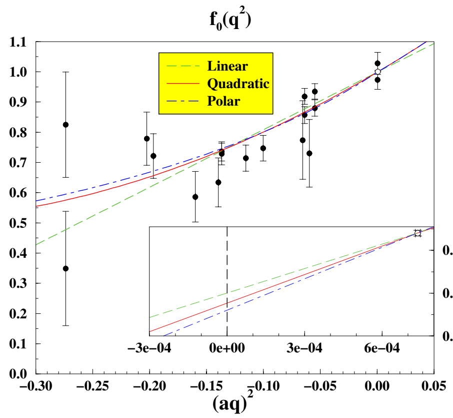

Fig. 2. The form factor $f_0(q^2)$ obtained from the double ratios (6) for $q^2 < q^2_{\max}$ (*full dots*) and from the double ratio (3) at $q^2 = q^2_{\max}$ (*open dot*), for the quark mass combination $k_s = 0.13390$ and $k_\ell = 0.13440$. The *dot-dashed*, *dashed* and *solid lines* correspond to the polar, linear and quadratic fits given in (7–9), respectively. The inset is an enlargement of the region around $q^2 = 0$

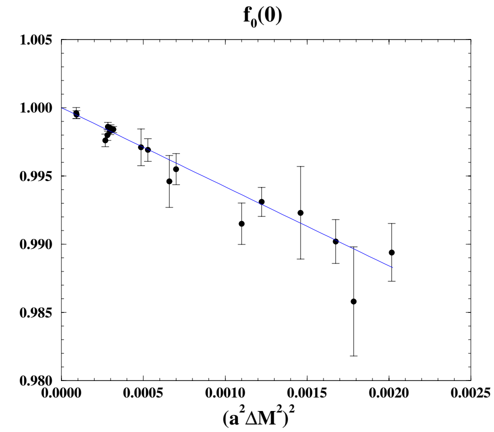

Fig. 3. Values of $f_0(0) = f_V(0)$, obtained from the quadratic fit (9), versus $(a^2\Delta M^2)^2$. The solid line is the result of the linear fit $f_0(0) = 1 - A\,(a^2\Delta M^2)^2$ where A is a mass-independent parameter

from which a determination of the ratio of the form factors $f_0(q^2)/f_V(q^2)$ is obtained. The advantages of the double ratios (6) are similar to those already pointed out for the double ratio (3), namely: i) a large reduction of statistical fluctuations; ii) the independence of the improved renormalization constant of the vector current, and iii) $R_i \to 1$ in the SU(3)-symmetric limit. We stress that the introduction of the matrix elements of degenerate mesons in (6) is crucial to largely reduce statistical fluctuations, because it compensates the different fluctuations of the matrix elements of the spatial and time components of the weak vector current.

Thanks to the ratios (6) the statistical uncertainties on $f_0(q^2)$ become $\simeq 5 \div 20\%$. The quality of the results is shown in Fig. 2 for one of the combinations of quark masses used in [6]. The points are paired because both $K \to \pi$ and $\pi \to K$ transitions are considered.

In order to extrapolate the scalar form factor to $q^2 = 0$ we have considered three different possibilities, namely a polar, a linear and a quadratic fit:

$$f_0(q^2) = f_0^{(pol.)}(0)/(1 - \lambda_0^{(pol.)}\, q^2)\,, \tag{7}$$

$$f_0(q^2) = f_0^{(lin.)}(0) \cdot (1 + \lambda_0^{(lin.)}\, q^2)\,, \tag{8}$$

$$f_0(q^2) = f_0^{(quad.)}(0) \cdot (1 + \lambda_0^{(quad.)}\, q^2 + c_0\, q^4)\,. \tag{9}$$

These fits are shown in Fig. 2 and provide values of both $f_0(0)$ and the slope λ_0, which are consistent with each other within the statistical uncertainties.

The results obtained for $f_0(0)$ agree well with a quadratic dependence on $a^2\Delta M^2$, as it can be seen from Fig. 3. This is expected from both lattice artifact contributions and physical SU(3)-breaking effects, which obey the AG theorem holding also in the quenched approximation [6].

Our results for the slope λ_0, extrapolated to the physical meson masses (using a linear dependence in the quark ones) and given in units of $M^2_{\pi^+}$, yield: $\lambda_0^{(pol.)} = 0.0122(22)$, $\lambda_0^{(lin.)} = 0.0089(11)$ and $\lambda_0^{(quad.)} = 0.0115(26)$. Our "polar" value $\lambda_0^{(pol.)}$ is consistent with the recent determination from KTeV $\lambda_0 = 0.01414 \pm 0.00095$ [9], obtained using a pole parameterization. Furthermore, the values obtained for the slope λ_V agree well with the inverse of the squared K^*-meson mass for each combination of the simulated quark masses. A simple linear extrapolation in terms of the quark masses to the physical values yields $\lambda_V = 0.026 \pm 0.002$ in units of $M^2_{\pi^+}$, which is consistent with the PDG value $\lambda_V = 0.028 \pm 0.002$ [2] as well as with the recent measurement from KTeV $\lambda_V = 0.02502 \pm 0.00037$ [9], obtained using a pole parameterization.

2.3 Extrapolation of $f_V(0)$ to the physical masses

In order to determine the physical value of $f_V(0)$ the lattice results of Fig. 3 should be extrapolated to the physical kaon and pion masses. The problem of the chiral extrapolation is substantially simplified if the AG theorem (holding also in the quenched approximation) is taken into account and if the leading (quenched) chiral logs are subtracted. Thus in [6] the following quantity is introduced

$$R(M_K, M_\pi) = \frac{1 + f_2^q(M_K, M_\pi) - f_V(0; M_K, M_\pi)}{(a^2\Delta M^2)^2} \tag{10}$$

where f_2^q represents the leading non-local contribution determined by pseudoscalar meson loops within quenched CHPT. The subtraction of f_2^q is a well defined procedure being finite and scale-independent. The value of f_2 in the unquenched case is known from [5], while its quenched counterpart is calculated in [6]. Thus, after the subtraction of f_2^q we expect that $R(M_K, M_\pi)$ receives large contributions from local operators in the effective theory. At the same time the quadratic dependence on $a^2\Delta M^2$, driven by the AG theorem, is already factorized out. Hence $R(M_K, M_\pi)$ is a quantity well suited for a smooth polynomial extrapolation in the meson masses. It turns out [6]

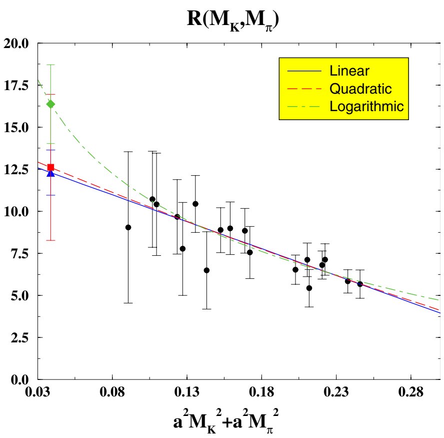

Fig. 4. Comparison among linear (11), quadratic (12) and logarithmic (13) fits of the ratio $R(M_K, M_\pi)$ as a function of $[a^2M_K^2 + a^2M_\pi^2]$. *Triangle, square* and *diamond* are the values of $R(M_K, M_\pi)$ extrapolated to the physical meson masses. For illustrative purposes we have chosen the case in which a quadratic fit in $q^2$is used to extrapolate f_0 to $q^2 = 0$

indeed that the dependence of $R(M_K, M_\pi)$ on the meson masses is well described by a simple linear fit:

$$R^{(lin.)}(M_K, M_\pi) = c_{11} + c_{12}[(aM_K)^2 + (aM_\pi)^2] \ , \quad (11)$$

whereas the dependence on ΔM^2 is found to be negligible. In order to check the stability of the results, quadratic and logarithmic fits have been also considered:

$$\begin{aligned} R^{(quad.)}(M_K, M_\pi) &= c_{21} + c_{22}[(aM_K)^2 + (aM_\pi)^2] \\ &\quad + c_{23}[(aM_K)^2 + (aM_\pi)^2]^2, \qquad (12) \\ R^{(log.)}(M_K, M_\pi) &= c_{31} + c_{32} \log[(aM_K)^2 + (aM_\pi)^2]. \ (13) \end{aligned}$$

In Fig. 4 it is shown that linear, quadratic and logarithmic functional forms provide equally good fits to the lattice data with consistent results also at the physical point.

Combining our estimate of $R(M_K, M_\pi)$ at the physical meson masses with the unquenched value of f_2 ($f_2 = -0.023$ [5]), we finally obtain [6]

$$f_V^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\rm stat} \pm 0.007_{\rm syst} \qquad (14)$$

where the systematic error comes mainly from the uncertainties in the functional dependence of f_0 on both q^2 and the meson masses. An estimate of quenched effects beyond $\mathcal{O}(p^4)$ is not included. Our value (14) compares with $f_V^{K^0\pi^-}(0) = 0.961 \pm 0.008$ [2], based on the LR result [5].

Using the (old) published $K_{\ell 3}$ data from [2], our value (14) implies $|V_{us}| = 0.2202 \pm 0.0025$, which still deviates by $\approx 2\sigma$ from the CKM unitarity relation. However, using only the recent high-statistics K_{e3} results of [10,11] one finds substantially higher values, $|V_{us}| = 0.2275 \pm 0.0030$ and $|V_{us}| = 0.2255 \pm 0.0025$, which are in good agreement with CKM unitarity. The experimental situation is expected to be further clarified in the near future when the results by KLOE [12] (on both charged and neutral modes) and by NA48 will become available.

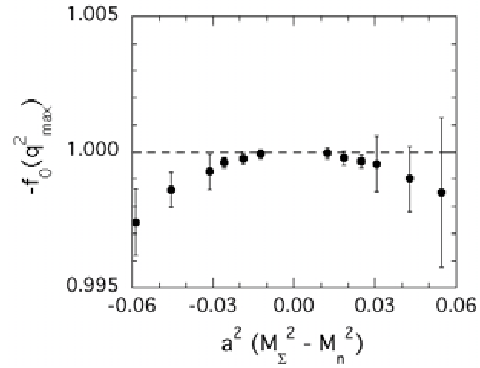

Fig. 5. Results for $f_0(q^2_{max})$ versus $a^2(M_\Sigma^2 - M_n^2)$

3 Semileptonic $\Sigma^- \to n\ell\nu$ decay

Semileptonic hyperon decays represent the "baryonic way" to a precise determination of $|V_{us}|$. In this section we present the results of a preliminary lattice study of $f_V(0)$ for the decay $\Sigma^- \to n\ell\nu$, carried out in [7].

The relevant matrix element of the weak vector current can be decomposed in terms of the following structures

$$\begin{aligned} < n|\overline{u}\gamma^\mu s|\Sigma^- > = \overline{u}_n(p') \Big\{ &\gamma^\mu f_V(q^2) - \frac{i\sigma^{\mu\nu}q_\nu}{M_n + M_\Sigma} f_2(q^2) \\ &+ \frac{q^\mu}{M_n + M_\Sigma} f_3(q^2) \Big\} u_\Sigma(p) \qquad (15) \end{aligned}$$

with $q = p - p'$. As in the case of $K_{\ell 3}$ decays, one introduces the scalar form factor $f_0(q^2)$:

$$f_0(q^2) = f_V(q^2) + \frac{q^2}{M_\Sigma^2 - M_n^2} \ f_3(q^2) \ , \qquad (16)$$

so that $f_0(0) = f_V(0)$. Note that the SU(3)-symmetric value of $f_V(0)$ is given by a Clebsch-Gordan coefficient, equal to (-1) for the $\Sigma^- \to n$ transition.

The value of $f_0(q^2)$ at $q^2_{\rm max} = (M_\Sigma - M_n)^2$ can be extracted using the double ratio (3), which now reads as

$$R_0 \xrightarrow[(t_y - t_x) \to \infty]{t_x \to \infty} \frac{\langle n|\bar{s}\gamma_0 u|\Sigma\rangle\langle\Sigma|\bar{u}\gamma_0 s|n\rangle}{\langle\Sigma|\bar{s}\gamma_0 s|\Sigma\rangle\langle n|\bar{u}\gamma_0 u|n\rangle} = [f_0(q^2_{\rm max})]^2. \ (17)$$

The results obtained for $f_0(q^2_{max})$ are shown in Fig. 5, where the high precision reached ($\lesssim 0.1\%$) can be seen.

Given the high accuracy reached at q^2_{max} as well as the closeness of the values of $q^2_{\rm max}$ to $q^2 = 0$, it is enough to study the q^2-dependence of $f_{0,V}(q^2)$ with an accuracy of $\approx 10 \div 20\%$ in order to reach the percent precision on $f_0(0)$. As in the case of mesons [6], the standard form factor analysis provides values of $f_V(q^2)$ quite well determined, whereas for $f_0(q^2)$ one has to resort to the double ratios (5), which give access to the quantity $f_0(q^2)/f_V(q^2)$. In Fig. 6 the values of $f_0(q^2)$ and $f_V(q^2)$ obtained for a specific combination of the quark masses used in [7], are reported. The points are paired because both $\Sigma^- \to n$ and $n \to \Sigma^-$ transitions are considered.

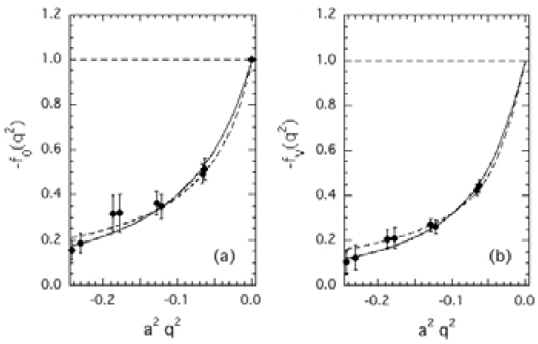

Fig. 6. Results for $f_0(q^2)$ (**a**) and $f_V(q^2)$ (**b**) versus a^2q^2. The *dashed* and *solid lines* are a monopole and a dipole fit to the lattice data, respectively [see (18)]

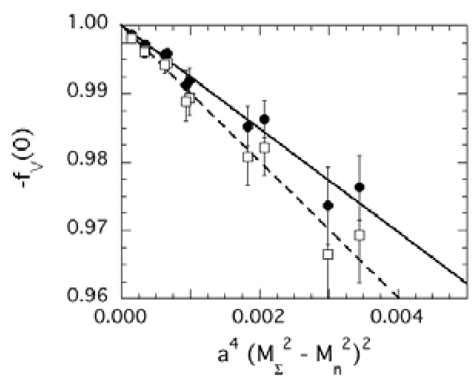

Fig. 7. Results for $f_V(0)$ versus $a^4(M_\Sigma^2 - M_n^2)^2$, obtained through a monopole (*open squares*) and a dipole fit (*full circles*) in q^2. The *dashed* and *solid lines* are linear fits, according to the AG theorem [4]

In order to get the values $f_0(0) = f_V(0)$ the results shown in Fig. 6 are fitted using both a monopole and a dipole functional forms

$$F^{(mon.)} = \frac{A}{1 - q^2/B}\ , \quad F^{(dip.)} = \frac{C}{(1 - q^2/D)^2}\ , \qquad (18)$$

which nicely describe the lattice data. The dipole parameter $\sqrt{D}$ agrees with the value predicted by pole dominance (the K^* meson mass) within $\simeq 15\%$ accuracy.

Finally in Fig. 7 we collect all the results obtained for $f_V(0)$ in [7]. They clearly exhibit the nice linear dependence expected from the AG theorem. Thus $SU(3)$-breaking effects are resolved with a good precision even within the limited statistics used in [7].

4 Conclusions

We have presented quenched lattice studies of the $K \to \pi$ and $\Sigma \to n$ vector form factors at zero-momentum transfer, $f_V(0)$. Our calculations are the first one obtained using a non-perturbative method based only on QCD, except for the quenched approximation. Our main goal is the determination of the SU(3)-breaking effects on $f_V(0)$, which is necessary to extract $|V_{us}|$ from both $K_{\ell 3}$ and hyperon decays. In order to reach the required level of precision we have employed the double ratio method originally proposed in [8] for the study of heavy-light form factors. We have found that this approach allows to calculate the scalar form factor $f_0(q^2)$ at $q^2 = q^2_{\max}$ with a statistical uncertainty well below 1% for both mesons and baryons.

A second crucial step is the extrapolation of the scalar form factor to $q^2 = 0$. This has been performed by fitting accurate results obtained using suitable double ratios of three-point correlation functions. The values of $f_V(0)$ obtained in this way are determined within the percent level of precision, which is the one required for a significant determination of $|V_{us}|$.

In case of $K_{\ell 3}$ decays the leading chiral artifacts of the quenched approximation, f_2^q, have been corrected for by means of an analytic calculation in quenched CHPT. After this subtraction, the lattice results can be smoothly extrapolated to the physical meson masses, obtaining

$$f_V^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}}\ , \qquad (19)$$

where the systematic error does not include an estimate of quenched effects beyond $\mathcal{O}(p^4)$.

The impact of our result on the determination of $|V_{us}|$ has been briefly addressed. Using the (old) published $K_{\ell 3}$ data from [2], we obtain $|V_{us}| = 0.2202 \pm 0.0025$, which still implies a $\approx 2\sigma$ deviation from the CKM unitarity relation. Using only the recent high-statistics K_{e3} results of [10,11] one finds substantially higher values, $|V_{us}| = 0.2275 \pm 0.0030$ and $|V_{us}| = 0.2255 \pm 0.0025$, which are in good agreement with CKM unitarity.

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